EE 2030 Linear Algebra Chi-chao Chao
Spring 2010

Homework Assignment No. 5
Due 10:10am, June 9, 2010
(This homework counts 1.5 times.)

`Reading:` Strang, Sections 5.3, 6.1, 6.2, 6.4, 6.5, 6.6, 6.7.

`Problems for Solution:`

1. Problem 6 in Problem Set 5.3 (p. 279) of Strang.

2. Problem 20 in Problem Set 5.3 (p. 280) of Strang.

3. Suppose that $\lambda_1, \lambda_2, \ldots, \lambda_n$ are eigenvalues of an $n \times n$ matrix $\boldsymbol{A}$. Show that

$$\lambda_1 + \lambda_2 + \cdots + \lambda_n = \text{trace}(\boldsymbol{A}).$$

4. Suppose that

$$G_{k+2} = \frac{1}{2}G_{k+1} + \frac{1}{2}G_k, \quad \text{for } k \geq 0$$

with $G_0 = 0$ and $G_1 = 1$.

   (a) Find a general formula for G_k, $k \geq 0$.
   (b) Find $\lim_{k\to\infty} G_k$.

5. Problem 18 in Problem Set 6.2 (p. 309) of Strang.

6. Find an orthogonal matrix $\boldsymbol{Q}$ that diagonalizes this matrix:

$$\boldsymbol{A} = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 0 & 0 \\ 2 & 0 & 0 \end{bmatrix}.$$

7. (a) Show that a real skew-symmetric matrix (i.e., $\boldsymbol{A}^T = -\boldsymbol{A}$) has pure imaginary eigenvalues. (Hint: The proof is similar to that for the eigenvalues of a real symmetric matrix.)

   (b) Show that an orthogonal matrix has all eigenvalues with $|\lambda| = 1$. (Hint: Consider $\|\boldsymbol{Ax}\|^2 = (\overline{\boldsymbol{Ax}})^T(\boldsymbol{Ax})$.)

   (c) Use (a) and (b) to find all four eigenvalues of

$$\boldsymbol{A} = \frac{1}{\sqrt{3}} \begin{bmatrix} 0 & 1 & 1 & 1 \\ -1 & 0 & -1 & 1 \\ -1 & 1 & 0 & -1 \\ -1 & -1 & 1 & 0 \end{bmatrix}.$$

   (Hint: Consider the trace of $\boldsymbol{A}$.)

8. Problem 10 in Problem Set 6.5 (p. 351) of Strang.

9. Problem 24 in Problem Set 6.5 (p. 352) of Strang.

10. Problem 5 in Problem Set 6.6 (p. 360) of Strang.

11. Problem 1 in Problem Set 6.7 (p. 371) of Strang.

12. Problem 17 in Problem Set 6.7 (p. 373) of Strang. Do parts (a), (b), and (c). Then find a 4×4 matrix $\boldsymbol{U}$, a 4×3 matrix $\boldsymbol{\Sigma}$, and a 3×3 matrix $\boldsymbol{V}$ in the singular value decomposition of $\boldsymbol{A}$:

$$\boldsymbol{A} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^T.$$